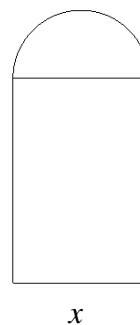


Name _____

**YOU MUST SHOW YOUR WORK. YOU MUST JUSTIFY YOUR ANSWERS.
CALCULATORS MAY NOT BE USED**

1. (10 points) A norman window has the shape of a rectangle surmounted by a semi-circle. If the perimeter of the window is 25 feet, express the area A of the window as a function of the width x of the window.



2. (10 points) If a bacteria population starts with 100 bacteria and doubles every three hours, then the number of bacteria after t hours is given by $n(t) = 100 \cdot 2^{t/3}$. Find the inverse of this function.
3. (10 points each) Find the following limits (**Choose only 2 out of the 3**):

a. $\lim_{x \rightarrow 7} \frac{3 - \frac{3}{x}}{x - 7}$ b. $\lim_{x \rightarrow -1} \frac{x + 1}{x + \sqrt{x + 2}}$ c. $\lim_{\theta \rightarrow \frac{\pi}{4}} \frac{1 - \tan \theta}{\sin \theta - \cos \theta}$

4. (10 points each) Find the following limits (**Choose only 2 out of the 3**):

a. $\lim_{x \rightarrow 3^-} \frac{|x - 3|}{x^2 - 2x - 3}$ b. $\lim_{x \rightarrow 3^+} \frac{4 - x}{x^2 - 9}$ c. $\lim_{x \rightarrow \infty} \frac{2x - 3}{\sqrt{x^2 - x - 2}}$

5. (10 points each) Find the following limits (**Choose only 2 out of the 3**):

a. $\lim_{x \rightarrow \infty} (-4\pi + e^{-4x})$ b. $\lim_{x \rightarrow 1^+} \ln(x^4 - 1)$ c. Use the Squeeze theorem: $\lim_{x \rightarrow 0} \sqrt[3]{x} \cos\left(\frac{2}{\sqrt[3]{x}}\right)$

6. Find $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h}$, where $f(x) = x^2 - 7x$

7. Choose (a) or (b) but **NOT** both

(a) Use the definition of continuity to find the value for A that will make the function continuous

for all x :
$$f(x) = \begin{cases} \frac{Ax - 2A}{x^2 - 5x + 6} & \text{if } x < 2 \\ -\frac{7}{10}x + 2A & \text{if } x \geq 2 \end{cases}$$

(b) Use the Intermediate Value Theorem to show that $f(x) = x^3 - 2$ and $g(x) = \sqrt{x}$ have at least one intersection point in the interval $[0, 2]$. {Hint: this is the same as showing that $f(x) - g(x)$ has at least one real root in this interval.

8. Sketch the graph of a function f that satisfies all of the following conditions:

$$\lim_{x \rightarrow 3^-} f(x) = -\infty$$

$$\lim_{x \rightarrow 3^+} f(x) = \infty$$

$$\lim_{x \rightarrow \infty} f(x) = 2$$

$$\lim_{x \rightarrow -\infty} f(x) = 2$$

$$f(-3) = 0$$

9. a) State the ϵ - δ definition of limit

b) Use the ϵ - δ definition of limit to prove $\lim_{x \rightarrow -1} (4 - 3x) = 7$