_____ **Calculus 1 Test 2** Section 1 or 2 (Circle one)

Name **March, 2004** **Chapter 3**

YOU MUST SHOW YOUR WORK. YOU MUST JUSTIFY YOUR ANSWERS.
CALCULATORS MAY NOT BE USED

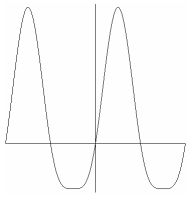
1. (10 points each) Compute the derivatives of the following functions, perform 1 line of simplification:

(a) $g(x) = \sin^{-1}(5x)$ (b) $h(x) = (e\pi)^{2x}$ (c) $L(t) = \dfrac{3t}{1-t^2}$

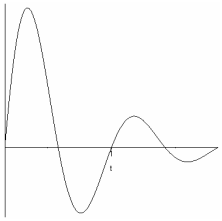
2. (10 points) Find the limit of $\lim\limits_{x\to -1} \dfrac{\sin(x+1)}{x^2-2x-3}$

3.(10 points) **Choose either (a) or (b), but not both**.

(a) Find all points on the graph of $f(x) = 2\sin x + \sin^2 x$ at which the tangent line is horizontal.

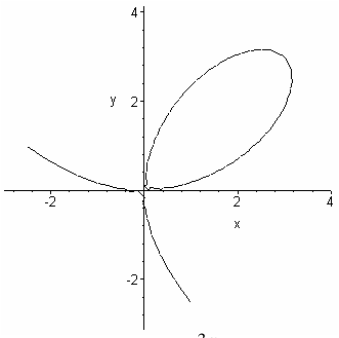


(b) Suppose the motion of a point on a spring on a shock absorber in Mr. Jeckel's car is given by the equation $s(t) = 2e^{-1.5t}\sin(2\pi t)$ where s is measured in centimeters and t in seconds. Find the velocity after t seconds.



4. (10 points) Consider the equation for the Folium of Descartes $x^3 + y^3 = 6xy$.

Use **implicit differentiation** to find $\dfrac{dy}{dx}$ in terms of x and y.



5. (10 points each) Use **logarithmic differentiation** to find the derivatives of $h(x) = (1+\sin \pi x)^{3x}$.

6. (10 points total) Find the tangent line to $g(x) = \ln(x+1) + 1$ at $x = 0$.

7. (10 points) **Choose (a) or (b) but not both**.

(a) Find the linear approximation of the function $f(x) = \dfrac{1}{(1+2x)^4}$ at $a = 0$. [$f(x) \approx f(a) + f'(a)(x-a)$]

(b) Poiseuille's Law states that when blood flows along a blood vessel, the flux $F = kR^4$, where k is a constant of proportionality, and R is the radius of the blood vessel. A partially clogged artery can be expanded by an operation called angioplasty, in which a balloon-tipped catheter is inflated inside the artery in order to widen it and restore the normal blood flow. Show that the relative change in F is about four times the relative change in R. $\left[\textit{The Relative change of a function f is given by } \dfrac{df}{f}\right]$.

8. (10 points) **Choose (a) or (b) but not both**.

(a) A street light is mounted at the top of a 15-ft tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

(b) As a spherical raindrop evaporates, its volume V changes at a rate proportional to its surface area A; i.e., $\dfrac{dV}{dt} = KA$, where K is called the constant of proportionality. Show that the rate of change of the radius is always constant. $V = \dfrac{4}{3}\pi r^3$ and $A = 4\pi r^2$.