DE - SUMMER 2003 FINAL EXAM - TAKE HOME PART

THIS IS AN EXAM. YOU ARE NOT TO GIVE OR RECEIVE HELP.

1. Find the general solution of $(1+x^2)y'' - 7xy' + 6y = 5x$.
2. Find the function y which satisfies the integral equation:
   $$y(t) = 2t^2 - \int_0^t y(t-\xi)\sinh(3\xi)d\xi$$
3. Find the Laplace Transform of the following function
   $$f(t) = \begin{cases} \cos \pi x, & \text{if } 0 < t < 3 \\ 0, & \text{if } t > 3 \end{cases}$$
4. Find the Fundamental Matrix solution of $\mathbf{x}' = \mathbf{Ax}$ where $\mathbf{A}$ is given by
   $$\mathbf{A} = \begin{pmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \\ 0 & 3 & 1 \end{pmatrix}$$
5. Find the unique solution of the following initial value problem: $\mathbf{x}' = \mathbf{Ax} + \mathbf{F(t)}$ where
   $$\mathbf{A} = \begin{pmatrix} 0 & 2 \\ 1 & -1 \end{pmatrix},$$
   $$\mathbf{F(t)} = \begin{pmatrix} e^t \\ 1 \end{pmatrix}$$
   and
   $$\mathbf{x(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}.$$