MATH 3391 EXAM 1 - PART I

FIND THE GENERAL SOLUTION

1. $y^{'} = \dfrac{xy - 3x}{x^2 + 1}$
2. $y' = \dfrac{2}{x}y + x^3$
3. $y^{''} + 5y^{'} + 7y = 0$
4. $y^{''} + 9y = e^{3x}$
5. $a_7 y^{(7)} + a_6 y^{(6)} + \cdots + a_1 y^{'} + a_0 y = 0.$
   Assume that $a_7\lambda^7 + a_6\lambda^6 + \cdots + a_1\lambda + a_0 = (\lambda^2 + 2\lambda + 4)^2(\lambda - 3)^2.$
6. $2x^2 \dfrac{d^2y}{dx^2} + 2x\dfrac{dy}{dx} + 8y = 8x$

PART II- OPEN BOOK

7. Under normal atmospheric conditions, the density of soot particles $N(t)$ in the atmosphere is modeled by the DE

$$\frac{dN}{dt} = -k_c N^2(t) + k_d N(t)$$

where k_c, called the **coagulation constant**, is a constant that relates how well particles stick together ; and k_d, called the dissociation constant, is a constant that relates to how well particles fall apart. **Both of these are constants which depend on temperature, pressure, particle size, and other external forces.** Find the general solution of this ODE by writing it as a Bernoulli equation and making the appropriate substitution.

8. Consider the linear ODE $a_3 x^2 \frac{d^3y}{dx^3} + a_2 x^2 \frac{d^2y}{dx^2} + a_1 x \frac{dy}{dx} + a_0 y = 0$. Show that the substitution $x = e^t$ transforms this ODE into a third order linear ODE with constant coefficients. Write the new ODE.

9. Show that the substitution $y = e^u$ transforms the following non-linear second order ODE to a second order *linear* equation for u. $y\frac{d^2y}{dt^2} - (\frac{dy}{dt})^2 - f(t)y^2 = 0$.

10. Find the unique solution of the following IVP:

$$y'' - 4y = 2t + 1, \; y(0) = 0, \; y'(0) = 1.$$