

Master's Exam: Real Analysis

March 20th 2004

*Solve **two** out of three problems. Show work.*

1. Let S be a dense set of real numbers \mathbb{R} . Show that if f is an extended real-valued function defined on \mathbb{R} , such that $\{x : f(x) < \alpha\}$ is measurable for every $\alpha \in S$, then f is measurable.
2. Prove that $f(x) = \frac{\sin x}{x}$ is Riemann integrable but not Lebesgue integrable on $[1, \infty)$.
3. State and prove the Lebesgue Dominated Convergence Theorem.