

Master's for Teachers Example Exam

Summer 2004

*Answer question 1 from section A and any **two** questions from section B.*

Section A

1. Several sixth graders are multiplying three-digit numbers in the following way:

$$\begin{array}{r} 123 \\ \times 645 \\ \hline 615 \\ 492 \\ 738 \\ \hline 1845 \end{array}$$

- (a) Identify the problem.
- (b) Explain the mathematics you would review with these students to help them correct the mistake.

Section B

2. Prove that if $f(x)$ is bounded on \mathbb{R} and $\lim_{x \rightarrow a} g(x) = 0$ then $\lim_{x \rightarrow a} f(x)g(x) = 0$.
3. If $[x]$ is the greatest integer function, does $\int_{-1}^4 [x] dx$ exist? If yes, evaluate it. If no, explain.

Please Turn Over

4. If $A = \begin{pmatrix} 1 & -1 & 4 \\ 3 & 2 & -1 \\ 2 & 1 & -1 \end{pmatrix}$, find all values of c such that

$$(A - cI) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

has a nontrivial solution.

5. (a) Suppose R is an integral domain. Prove that the only solutions to the equation $x^2 = 1$ are $x = 1$ and $x = -1$.
- (b) Let R be the ring $\{0, 1, 2, 3, 4, 5, 6, 7\}$ with addition and multiplication defined modulo 8. How many solutions does the equation $x^2 = 1$ have in R ?
- (c) Give an example of a ring in which the equation $x^2 = 1$ has only one solution.

6. The letters of the word

MISSISSIPPI

are arranged in random order. Let X be the numbers of I's that are immediately followed by an S. Find $\mathbb{E}(X)$.