Topology
Ph.D. qualifying exam

October 26, 2001

Name:

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1. Let R be the set of all real numbers with the countable complement topology, Γ, i.e. $A \in \Gamma$ if and only if $A = \emptyset$ or $R \setminus A$ is countable. Answer the following two questions and justify carefully your answers.
   (a) Is (R, Γ) metrizable?
   (b) Identify the subsets of R that are simultaneously connected and compact.
2. Let R be the set of all real numbers with the usual topology and R^N the infinite product of countably many copies of R with the product topology. Does R^N have a countable dense subset? Explain your answer.
3. Let (X, d) be a compact metric space and $f : X \to X$ a continuous map. Show **only one** of the following two statements:
   (a) If f is a contraction (i.e. there exists $0 < \alpha < 1$ such that for every x and y in X $d(f(x), f(y)) < \alpha\, d(x, y)$) then there exists a unique $x \in X$ such that $f(x) = x$.
   (b) If f is an isometry (i.e. for x and y in X, $d(f(x), f(y)) = d(x, y)$) then f is surjective.
4. Solve **only one** of the following two problems:
   (a) Give an example showing that the product of normal spaces is not necessarily normal.
   (b) Every metrizable topological space is normal.
5. Choose **only one** of the following two problems and state all the results you use:
   (a) Show that the fundamental group of the figure eight is not abelian.
   (b) Show that S^2 (2-sphere) and RP^2 (projective 2-space) are topologically distinct.