PRACTICE FINAL EXAM
MATH 3120 , FALL 2006

Problem 1: [25p] Consider the following differential equation:

$$2(x-1)y' = 3y\,.$$

1) [10p] Solve this differential equation.

2) [15p] Use power series method to write a power series solution of the given differential equation (without using the solution found above). Find the radius of convergence of this power series.

Problem 2: [15p] Verify that the following differential equation is exact.

$$(y^2 + x^2 - 2x + 3)dx + (2xy - y^2 + 10)dy = 0\,.$$

The, use this information to solve it.

Problem 3: [20p] Consider the following non-homogeneous linear differential equation:

$$y^{(4)} + y^{(3)} - 5y^{(2)} + y' - 6y = 3x + 1\,.$$

1) [10p] Find the general solution of the associated homogeneous equation. (Hint: Use the fact that $r^4 + r^3 - 5r^2 + r - 6 = (r-2)(r+3)(r^2+1)$.)

2) [10p] Find a particular solution y_p of the given non-homogeneous linear differential equation, and then find its general solution.

Problem 4: [25p] Use egenvalues/eigenvectors method to find the solution of the following homogeneous system with given initial conditions

$$\begin{aligned} x_1' &= 2x_1 + 4x_2\,, \quad x_1(0) = 1 \\ x_2' &= 3x_1 - 2x_2\,, \quad x_2(0) = 0\,. \end{aligned}$$

Problem 5: [25p] Consider the following matrices:

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = A + B$$

1) [5p] Compute e^{At} , for all real numbers t .

2) [10p] Compute e^{Bt} , for all real numbers t .

3) [5p] Compute e^{Ct} , for all real numbers t .

4) [5p] Find a particular solution of the system whose coefficient matrix is C , satisfying the initial condition $\vec{x}(0) = \begin{bmatrix} -1 \\ 4 \\ 0 \end{bmatrix}$

Problem 6: [20p] Consider the following second order IVP:

$$y'' + 6y' + 18y = 0\,, \quad y(0) = 2\,, y'(0) = -3\,.$$

1) [10p] Let $Y(s) = \mathcal{L}(y(t))$, where $y(t)$ is the solution of this IVP. Determine $Y(s)$, only. Do not take the inverse transform!

2) [10p] Use partial fractions to determine

$$\mathcal{L}^{-1}\left(\frac{2s+9}{s^2+6s+18}\right).$$

Problem 7: [70p] Compute each of the following:

1)[15p] $\mathcal{L}((t^2+4)\sin(5t))\,.$

2)[15p] $\mathcal{L}^{-1}\left(\frac{e^{3s}}{s^2-2s+1}\right).$

3)[10p] $\mathcal{L}^{-1}\left(\frac{3s^2+5s+2}{(s^2+1)(s+5)}\right).$

4)[15p] $\mathcal{L}^{-1}(\ln(s^2+2s+5))\,.$

5)[15p] $(f*g)(t)$ if $\mathcal{L}(f(t)) = \frac{e^{-3s}}{s}\,,\ \mathcal{L}(g(t)) = \frac{1}{s^3}\,.$