

There are 110 points on this exam. You need 100 points to get a perfect score. Show **ALL** work. Justify your steps using **COMPLETE** sentences. GOOD LUCK!

1. (15 points) Graph the function $f(x) = -3x^2 - 3x + 1$ and find its maximum value.

2. (20 points) Solve the equations:

$$(a) \frac{x}{x-1} - \frac{1}{x+1} = \frac{3}{x^2-1}, \quad (b) \sqrt{x^2+x} = x+1, \quad (c) |2x-1| = x+1.$$

3. (15 points) Solve the inequalities:

$$(a) -5 < -x + 2 < 3, \quad (b) |4 - 3x| > 10.$$

4. (20 points) Let $g(x) = x(x-1)^2(x+4)$.

(a) Find the zeros of g , their multiplicities, and determine the sign of g .

(b) Sketch the graph of $g(x)$.

5. (10 points) Find a polynomial p with rational coefficients such that $p(\sqrt{2}) = 0$ and $p(i) = 0$ (where $i = \sqrt{-1}$).

6. (20 points) Solve the inequalities:

$$(a) x^2 > 3x - 2, \quad (b) \frac{x+1}{x-2} \geq 3, \quad (c) \frac{x^2-1}{x(x+2)} > 0.$$

7. (10 points) Decompose the rational function

$$f(x) = \frac{1}{x(x+1)^2}$$

into partial fractions.