

MATH 8026 Spring 2008
ERGODIC RAMSEY THEORY

Instructor: Nikos Frantzikinakis, frantzikinakis@gmail.com.

Class Hours: TR 2:40-4:05 PM, Dunn Hall 201.

Office Hours: To be announced.

Prerequisite: Measure Theory.

Course grades: Grades will be assigned on the basis of the homework (4-5 assignments total).

TOPICS

Introduction: Examples of measure preserving transformations, recurrence, Furstenberg's correspondence principle, translation of results in Ramsey theory to ergodic theory.

Ergodicity: Examples of ergodic transformations, Von Neumann (mean) and Birkhoff (pointwise) ergodic theorem, applications to normal numbers, uniform distribution, continued fractions, generalization of law of large numbers.

Weak mixing: Examples, equivalent characterizations of weak mixing via the spectral theorem for unitary operators, multiple weak mixing.

Fundamentals: Lebesgue spaces, isomorphism, conjugacy, factors, decomposition of a measure over a factor algebra, ergodic decomposition, Rokhlin towers, eigenvalues, discrete spectrum theorem of Halmos and Von Neumann, Kronecker factor.

Continuous transformations: Invariant measures for continuous transformations on compact spaces, extreme points, mutual singularity of ergodic measures, unique ergodicity, Weyl's uniform distribution theorem,.

Warmup applications to Ramsey theory: Sárközy's theorem, and Roth's theorem on arithmetic progressions of length three.

Main applications to Ramsey theory: Szemerédi's theorem on arithmetic progressions, and the polynomial Szemerédi theorem of Bergelson and Leibman.

Further topics (time permitting): An overview of recent developments, the role of nilsystems in proving multiple recurrence results in ergodic theory.

Recommended texts in ergodic theory

- (1) An introduction to ergodic theory, P. Walters
- (2) Ergodic theory, K. Petersen

Recommended text in Ramsey theory

- (1) Ramsey theory, R. Graham, B. Rothschild, J. Spencer

Recommended texts in ergodic Ramsey theory

- (1) Recurrence in ergodic theory and combinatorial number theory, H. Furstenberg
- (2) Elemental methods in ergodic Ramsey theory, R. McCutcheon

Recommended articles

- (1) H. Furstenberg. Ergodic behavior of diagonal measures and a theorem of Szemerédi on arithmetic progressions. *J. d'Analyse Math.*, 71, (1977), 204-256.
- (2) H. Furstenberg, Y. Katznelson, D. Ornstein. The ergodic theoretical proof of Szemerédi's theorem. *Bull. Amer. Math. Soc. (N. S.)*, 7, (1982), no. 3, 527-552.
- (3) V. Bergelson, S. Leibman. Polynomial extensions of Van der Waerden's and Szemerédi's theorems. *J. Amer. Math. Soc.*, 9, (1996), 725-753.
- (4) V. Bergelson, Ergodic Ramsey theory - an update (survey article). *Ergodic theory of \mathbb{Z}^d -actions*, London Math. Soc. Lecture Note Series, 228, (1996), 1-61.