

**2701 Homework Assignment 2****Fall 2007****Due Tuesday 9th October.**

1. Are the following statements true or false. Explain why.
  - (a)  $\forall n: \exists p: (p > n \text{ and } p \text{ is prime})$ .
  - (b)  $\exists p: \forall n: (p > n \text{ and } p \text{ is prime})$ .
  - (c)  $\exists n: \forall p: (p > n \text{ and } p \text{ is prime})$ .
2. For each of the statements in question 1, write the negation of the statement (1) in symbols without the  $\sim$  sign (e.g., the negation of  $x > y$  should be written  $x \leq y$  etc.), and (2) as a simple English statement.
3. Prove by induction that

$$\frac{1}{1 \times 2} + \frac{1}{2 \times 3} + \frac{1}{3 \times 4} + \cdots + \frac{1}{(n-1) \times n} = 1 - \frac{1}{n}$$

4. Show that the product of three consecutive integers is always divisible by 6.
5.
  - (a) Find the gcd of 5394 and 7735.
  - (b) Find integers  $x$  and  $y$  such that  $5394x + 7735y = \gcd(5394, 7735)$ .
  - (c) Find a solution to the modular 'equation'  $5394x \equiv 1 \pmod{7735}$ .