7261 Homework Assignment 1 Fall 2006

Due Friday September 15.

1. Define a binary operation $\star$ on $\mathbb{Q}$ by $a \star b = a + b - ab$. Is $\mathbb{Q}$ a group under this operation? Explain.

2. Let G be a group of odd order. Show that for each $g \in G$ there is a *unique* $x \in G$ with $x^2 = g$.

3. Let G be a cyclic group of order n. Show every subgroup of G is cyclic with order d for some $d \mid n$. Conversely, show that for each d with $d \mid n$ there is a unique subgroup of this order.

4. Determine all the subgroups of D_6. For each one, determine whether or not it is normal.

5. Let H be a proper subgroup of a *finite* group G. Show that G is not the union of the conjugates xHx^{-1} of H. [Hint: Show that $xH = yH$ implies $xHx^{-1} = yHy^{-1}$ and count elements.]