

Due Friday September 29.

1. Suppose $K \trianglelefteq G$ and $m = [G : K]$ is relatively prime to $|K|$. Show that $K = \{x^m : x \in G\}$. Is this true if K is not assumed to be normal?
2. Show that if $G/Z(G)$ is cyclic then G is abelian. Deduce that if G is non-abelian then $Z(G)$ is strictly contained in some abelian subgroup of G .
3. Show that if p is prime, $H \leq S_p$ and H contains a transposition and a p -cycle then $H = S_p$. Give an example that shows that this may fail when p is not prime.
4. Let $\sigma \in A_n$. Suppose C is the conjugacy class of σ in S_n and C' is the conjugacy class of σ in A_n . Show that $C = C'$ if and only if either
 - (a) The cycle type of σ includes an odd cycle (cycle of even length), or
 - (b) The cycle type of σ includes at least two cycles of the same length (possibly 1-cycles).

Show further that if these conditions do not hold then C is the disjoint union of C' and C'' where C'' is another conjugacy class of A_n .

5. Let p be a prime.
 - (a) How many subgroups are there of $C_p \times C_p$ of order p ?
 - (b) How many subgroups are there of $C_p \times C_p \times C_p$ of order p^2 ?