Math 2000 Test 3 Example Solutions Spring 2004

1. The lifespan of a lightbulb is random with a probability distribution given by the table below.

Hours	0–999	1000–1999	2000–2999	3000–3999	≥4000
Probability	0.11	0.26	0.22	0.17	??

   (a) What is the probability of a lightbulb lasting at least 4000 hours?
   $1 - (0.11 + 0.26 + 0.22 + 0.17) = \mathbf{0.24}$

   (b) What is the probability of a lightbulb lasting less than 3000 hours?
   $0.11 + 0.26 + 0.22 = \mathbf{0.59}$

2. A bag of 30 M&M's contains 10 red, 10 green, and 10 yellow M&M's. If you pick two M&M's from the bag at random, what is the probability that they will be the same color?

   Sample space $\#S = 30 \times 29 = 870$. Event $\#E = 30 \times 9 = 270$ (2nd M&M must be one of the 9 remaining M&M's that has the same color as the 1st).
   Probability $= \mathbf{\frac{270}{870}} = \mathbf{\frac{9}{29}} = \mathbf{0.31}$

3. A bank allocates PIN numbers consisting of 4 digits with no repetitions, each such number occurring with equal probability. What is the probability that a PIN does not contain the number 3.

   Number of outcomes is $\#S = 10 \times 9 \times 8 \times 7 = 5040$.
   Number of outcomes with no 3 is $\#E = 9 \times 8 \times 7 \times 6 = 3024$.
   Probability is $\frac{3024}{5040} = \mathbf{0.6}$.

4. In a game costing \$9 to play, two dice are rolled.
   If they both roll a 5 or a 6, you win \$72. Otherwise you lose.

   a) Find the probability that you win if you play the game.
   Number of outcomes is $\#S = 6 \times 6 = 36$.
   Number of outcomes with both rolls 5 or 6 is $\#E = 2 \times 2 = 4$.
   Probability is $\frac{4}{36} = \mathbf{\frac{1}{9}}$.

   b) Show that your mean profit in a single round of the game is $-\$1.00$.
   Winning probability $\frac{1}{9}$, profit $72 - 9 = \$63$. Losing probability $1 - \frac{1}{9} = \frac{8}{9}$, profit $-\$9$.
   Mean profit $(\frac{1}{9})(63) + (\frac{8}{9})(-9) = 7 - 8 = \mathbf{-1}$.

   The standard deviation in a single round is \$22.60.

   c) What can be said about the distribution of the average profits over a course of 510 games?
   Approximately normal, mean $-\$1$, standard deviation $\frac{22.60}{\sqrt{510}} = \$1.00$.

   d) What is the approximate probability of making a loss over 510 games?
   Probability of being less than 1 standard deviation from mean $= 68\%$.
   Probability of being more than 1 standard deviation from mean $= 100 - 68 = 32\%$.
   Probability of being more than 1 standard deviation above mean $= 32/2 = 16\%$.
   Probability of loss is $100 - 16 = \mathbf{84}\%$.

5. A manufacturer makes ball bearings. The diameter of the ball bearings should be distributed normally with mean 1.000cm and standard deviation 0.006cm. Samples of 4 balls are taken and the average diameter measured. Give control limits (3 standard deviations above and below the mean) for this average.

   Average diameter is normal, mean 1.000 cm, standard deviation $\frac{0.006}{\sqrt{4}} = 0.003$ cm.
   Control limits $1.000 \pm 3(0.003) = 1.000 \pm 0.009$ cm, or **0.991 cm to 1.009 cm**.

6. In an opinion poll, 360 out of a simple random sample of 600 people answer Yes to a question. Give a 95% confidence interval for the percentage of the population which would answer yes.

   Sample proportion $\hat{p} = \frac{360}{600} = 60\%$. This is approximately normal with mean p and standard deviation about $\sqrt{\frac{60(100-60)}{600}}\% = \sqrt{4}\% = 2\%$.
   95% Confidence interval is $(\mathbf{60 \pm 2(2)})\% = (\mathbf{60 \pm 4})\%$.

7. Suppose you know that the average income is \$30,000 and the standard deviation of income is \$10,000. It is **not** correct to say that 95% of the population has income between \$10,000 and \$50,000. Why?

   The range is ±2 standard deviations from the mean, but the probability distribution of incomes is *not* given by a normal distribution, so this does not necessarily include 95% of the population. (68–95–99.7 rule only applies to normal distributions.)

8. The Postnet code consists of long and short bars. The code starts and ends with a single long bar, and between them groups of 5 bars represent digits. The last digit is a check digit chosen so the sum of all digits is divisible by 10. For example

   6 0 1 9 4

   represents the number 6019 with check digit chosen so that $6+0+1+9+4 = 20$ is divisible by 10.

   1
   2
   3
   4
   5
   6
   7
   8
   9
   0

   a) In the following, a single error has occurred. What is the correct number represented?

      6 8 x 6 8

      Check sum $6 + 8 + x + 6 + 8 = 28 + x$ must be divisible by 10, so $x = 2$ and original data is **6826**.

   b) The check digit is unchanged if two of the digits in the number are swapped. Why is this not likely to be a serious concern?

      Answer: **Scanning machines are unlikely to swap two digits of the code.**