Math 2000 Test 3 Solutions Spring 2004

1. (4 points) The following table gives the probabilities of rolling a number on a weighted die.

Number	1	2	3	4	5	6
Probability	0.10	0.16	0.16	0.16	0.16	??

   (a) What is the probability of rolling a 6?
   $1 - (0.10 + 0.16 + 0.16 + 0.16 + 0.16) = 1 - 0.74 = \mathbf{0.26}$.

   (b) What is the probability of rolling an odd number?
   $0.10 + 0.16 + 0.16 = \mathbf{0.42}$

2. (5 points) A large basket of fruit contains 3 oranges, 2 apples, and 5 bananas.

   (a) If one piece of fruit is chosen at random, what is the probability of getting an orange or a banana?
   $\#S = 10$ (there are ten fruit in total)
   $\#E = 3 + 5 = 8$ (eight are either oranges or bananas)
   Probability $= \frac{8}{10} = \mathbf{0.8}$.

   (b) If two pieces of fruit are chosen at random, what is the probability of getting an orange *and* a banana?
   $\#S = 10 \times 9 = 90$ (choose 2 different fruit)
   $\#E = 3 \times 5 + 5 \times 3 = 30$ (1st=orange, 2nd=banana, *or* 1st=banana, 2nd=orange)
   Probability $= \frac{30}{90} = \frac{1}{3} = \mathbf{0.33}$.

3. (4 points) My bike lock has the number 2361. What is the probability that if I buy another bike lock (also with a 4 digit code taking values between 0 and 9 where repeats are possible) that the digits will be the same as 2361 (possibly in a different order? e.g. 1623 or 2163...)

   $\#S = 10 \times 10 \times 10 \times 10 = 10000$ (each digit can be one of 10 values)
   $\#E = 4 \times 3 \times 2 \times 1 = 24$ (1st digit can be any of $\{2, 3, 6, 1\}$, 2nd digit is any of these, but not the same as 1st digit etc.)
   Probability $= \frac{24}{10000} = \mathbf{0.0024}$.

4. (8 points) A game consists of rolling two dice. If you roll a double (same number on both dice) then you win \$16 otherwise you lose \$2.

   a) What is the probability of winning?
   $\#S = 6 \times 6 = 36$, $E = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6)\}$, $\#E = 6$.
   Probability $= \frac{6}{36} = \mathbf{\frac{1}{6}}$

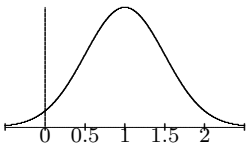
   b) Show that the mean winnings per game is \$1.
   Mean $\mu = (\mathbf{16})\frac{\mathbf{1}}{\mathbf{6}} + (\mathbf{-2})\frac{\mathbf{5}}{\mathbf{6}} = \frac{\mathbf{16-10}}{\mathbf{6}} = \mathbf{1}$

	Win	Lose
Winnings	16	-2
Probability	$\frac{1}{6}$	$1 - \frac{1}{6} = \frac{5}{6}$

   The standard deviation of the winnings in one game is about \$6.71.

   c) What can be said about the distribution of the winnings per game if 180 games are played?
   The average winning per game will have an approximately normal distribution with mean \$1 and standard deviation $\frac{\sigma}{\sqrt{\mathbf{n}}} = \frac{\mathbf{6.71}}{\sqrt{\mathbf{180}}} = \mathbf{\$0.50}$**.**

d) What is the probability of making a loss after playing 180 games?
Probability of loss is **2.5%**. (Must be at least 2 standard deviations below mean, but there is $100\% - 95\% = 5\%$ chance of being at least 2 s.dev above or below mean).



5. (4 points) A scientist measures the lengths of 100 adult earthworms. The average length is 4.92 cm with a standard deviation of 0.80 cm. Give a 95% confidence interval for the average length of an earthworm.

   Standard deviation for the average of 100 earthworms is $\frac{\sigma}{\sqrt{n}} = \frac{0.80}{\sqrt{100}} = 0.08$
   Confidence interval is $\mathbf{4.92 \pm 2(0.08) = 4.92 \pm 0.16}$ **cm.**

6. (6 points) In an opinion poll a simple random sample of 300 people were asked a question and 75 answered 'Yes'.

   a) If many similar opinion polls were conducted, what can you say about the distribution of the results obtained?
   Normal distribution with mean p and standard deviation $\sqrt{\frac{\mathbf{p(1-p)}}{\mathbf{300}}}$ where p is the proportion of the population who would answer 'Yes'.

   b) Give a 95% confidence interval for the percentage of the population who would answer yes to this question.
   Assuming the sample proportion $\hat{p} = \frac{75}{300} = 0.25 = 25\%$ is close to p, standard deviation is about $\sqrt{\frac{25 \times 75}{300}} = 2.5\%$. Confidence interval is $\mathbf{\hat{p} \pm 2(s.dev) = (25 \pm 5)\%}$.

7. (6 points) ISBN numbers use weights 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 respectively for the digits and modulus 11.

   a) An ISBN number is given as 0-7067-3709-2. Is this correct? Why?
   $\mathbf{10(0) + 9(7) + 8(0) + 7(6) + 6(7) + 5(3) + 4(7) + 3(0) + 2(9) + 1(2) = 210}$.
   This is not divisible by 11, so the number is **not correct**.

   b) What would be the check digit for the ISBN number 0-3128-2010-?
   $\mathbf{10(0) + 9(3) + 8(1) + 7(2) + 6(8) + 5(2) + 4(0) + 3(1) + 2(0) + 1(x) = 110 + x}$
   should be divisible by 11. But 110 is divisible by 11, so we can take $\mathbf{x = 0}$.

8. (3 points) ISBN codes (as described in question 7) can detect

   a) single-digit errors — (always) / sometimes / never (circle one)

   b) swapped digits — (always) / sometimes / never (circle one)

   c) insertions/deletions — (always) / sometimes / never (circle one)

Grading Scale: A: 31–40, B: 25–30, C: 19–24, D: 15–18, F: 0–14.