Math 2000 Test 4 Example Solutions Spring 2004

1. a) Suppose you wished to find a binary linear error-correcting code which correct up to 3 errors reliably. What is the minimum weight needed for this code? Explain.

   The maximum number of errors that can be reliably corrected is $(w-1)/2$. Hence $(w-1)/2 = 3$, so $w = \mathbf{7}$.

   b) Now suppose you only needed to detect 3 errors. What is the minimum weight needed now?

   The maximum number of errors that can be reliably corrected is $w-1$. Hence $w-1=3$, so $w=\mathbf{4}$.

2. Consider the following binary linear code with 3 digits + 3 check digits [Each row gives weights, and each weighted sum should be even.]

a_1	a_2	a_3	c_1	c_2	c_3
1	1	0	1	0	0
0	1	1	0	1	0
1	0	1	0	0	1

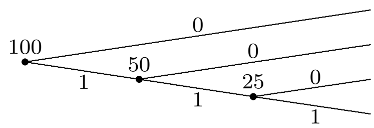
   a) Find the code for 011.

   Append c_1, c_2, c_3 so that check sums are even. Answer **011101**.

   b) A code is given as 100011. Assuming at most one error occurred, what was the original 3-digit number?

   Check sums are 1,1,2. The only 1 bit change that will make all check sums even is 1**1**0011. The original 3-digit number is **110**.

3. Construct an efficient binary code for the letters A, B, C, D where the frequencies are given by



Letter	Frequency	Code
A	50%	0
B	25%	10
C	15%	110
D	10%	111

   Translate ABACABAD into this code. **01001100100111**

4. Convert 30 meters per second into miles per hour. [1 mile = 1609 meters].

   1 mile = 1609 meters, 1 hour = 60×60 seconds = 3600 seconds. So $30\frac{1\text{ meter}}{1\text{ second}} = 30\frac{(1/1609)\text{mile}}{(1/3600)\text{hour}} =$ **67.12 mph**

5. A 180mm diameter circular silicon wafer can be used to produce 162 microprocessors. How many microprocessors do you expect can be made using a 300mm silicon wafer?

   The Linear scaling factor is $L = \frac{300}{180}$. Area scales as L^2, so number of processors is $162\left(\frac{300}{180}\right)^2 = \mathbf{450}$.

6. A 1lb weight has a diameter of 2in. What do you expect the diameter of an 8lb weight to be if it is the same shape?

   Weight scales as L^3, so $8 = 1L^3$ and $L = 2$. The diameter scales as L, so diameter is $2L =$ **4in**.

7. Which is a better deal, a 10in pizza for \$6 or a 12in pizza for \$8? Why?

   Linear scaling factor $L = \frac{12}{10} = 1.2$. Amount of pizza is likely to be proportional to area (assuming constant thickness of pizza). Hence one would expect the larger pizza price to be $\$6 \times L^2 = \$6 \times 1.2^2 = \$8.64$, so larger pizza is better deal.

8. If a quantity A is 25% more than B, what percentage is B less than A?

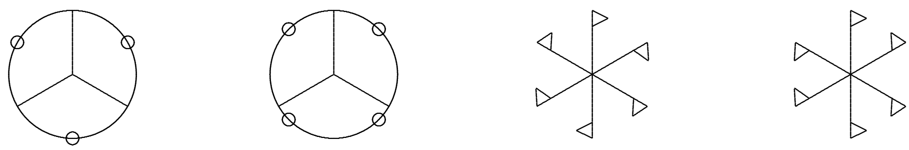
   1) **<u>20%</u>** 2) 25% 3) 33%

   $A = (1 + \frac{25}{100})B = 1.25B$, hence $B = \frac{1}{1.25}A = 0.80A = (1 - \frac{20}{100})A$. So B is 20% less than A.

9. A scale model is made of an airplane that is one ninth the linear dimensions of the original. Assuming is is geometrically similar and made of the same materials, at what proportion of the speed of the original can it fly?
   [Hint: (Pressure on wing) $\propto$ (Speed)^2.]

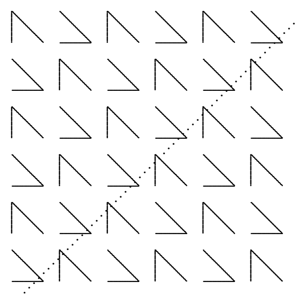
   $L = \frac{1}{9}$. Pressure $= \frac{\text{Weight}}{\text{Area}} \propto \frac{L^3}{L^2} = L$. Hence (Speed)$^2 \propto L$, or Speed $\propto \sqrt{L}$. $\sqrt{L} = \frac{1}{3}$, so one would expect it to fly at **one third the speed**.

10. Give the symmetry type (e.g., d4, c5) for each of the following figures and draw on each diagram all lines of reflectional symmetry (if there are any).



(a) **d3** (b) **d1** (c) **c6** (d) **d3**

11. Which of the following symmetries does the following pattern have?



   a) Reflection (draw line) **<u>No</u>**
   b) Glide reflection (draw line) **<u>Yes</u>**
   c) (Non-trivial) Rotation (mark center) **<u>No</u>**