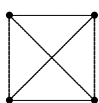
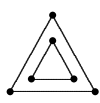


# Math 2000      Final Example Solutions      Spring 2004

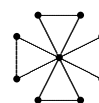
1.



(A)



(B)



(C)

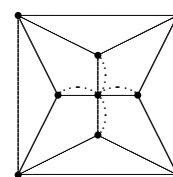
For each of the three graphs above state (Yes/No) whether

- |                                 |                |               |                |
|---------------------------------|----------------|---------------|----------------|
| a) it is connected              | (A) <u>Yes</u> | (B) <u>No</u> | (C) <u>Yes</u> |
| b) it has an Eulerian circuit   | (A) <u>No</u>  | (B) <u>No</u> | (C) <u>Yes</u> |
| c) it has a Hamiltonian circuit | (A) <u>Yes</u> | (B) <u>No</u> | (C) <u>No</u>  |

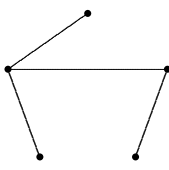
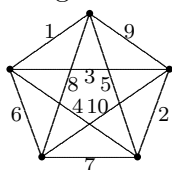
2. a) Does the graph shown have an Euler circuit? Explain.

**No, not every vertex has even valency.**

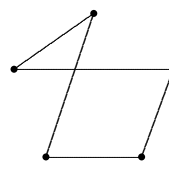
- b) If not, duplicate as few edges as possible so that it does have an Euler circuit.



3. The diagram shown gives the distances between five towns.



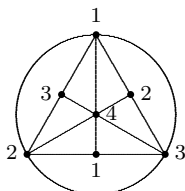
a) MST



b) TSP

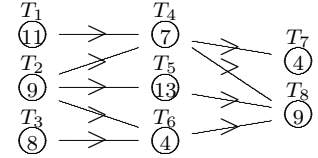
- a) Find a minimum spanning tree (MST).  
 b) Find a solution to the traveling salesman problem (TSP) using the sorted edges algorithm.  
 c) Which solution would be most useful for delivering mail? MST / TSP  
 d) Which would be most useful for building a cable TV network? MST / TSP

4. Color the vertices of the following graph with the minimum number of colors so that neighboring vertices receive distinct colors. How many colors are required?



\_\_\_\_\_ **4** \_\_\_\_\_ colors

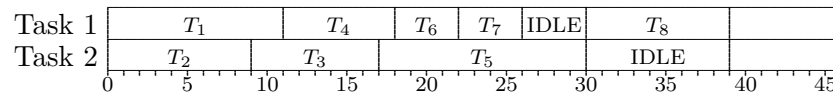
5. A complex process is composed of 8 tasks numbered  $T_1$  through  $T_8$ .  $T_1$  and  $T_2$  need to have finished before  $T_4$  starts;  $T_2$  needs to have finished before  $T_5$  starts;  $T_2$  and  $T_3$  need to have finished before  $T_6$  starts;  $T_4$  needs to have finished before  $T_7$  can start and  $T_8$  can only start when  $T_4$ ,  $T_5$  and  $T_6$  are finished. The tasks take respectively 11,9,8,7,13,4,4 and 9 units of time.



a) Draw a graph representing this information.

b) Give the critical path(s).  **$T_2, T_5, T_8$**

c) Schedule the process using two processors and priority list  $T_1, T_2, T_3, \dots, T_8$ .



d) Could this time be improved if more processors were used? What would be the shortest possible time if many processors were used? (You do **not** have to compute any other schedules).

**Yes. If enough processors were used the tasks could be completed in time 31 since this is the length of the critical path.**

6. A stuffed toy manufacturer can make cuddly mice and cats. Two items are in short supply: noses for the toys (the toys have the same noses) and stuffing material. The total number of noses is 100. There are 150 packs of stuffing material. Each cat uses 2 packs of stuffing material, but a mouse only uses 1 pack.

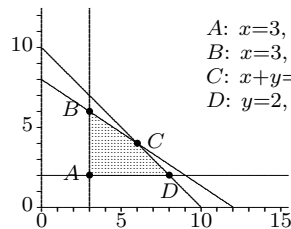
a) Introduce variables describing the problem.

**Let  $x$  be the number of cuddly mice and  $y$  the number of cats.**

b) List all the inequalities needed to specify the feasible set of solutions.

$$x + y \leq 100, \quad x + 2y \leq 150, \quad x \geq 0, \quad y \geq 0.$$

7. Maximize  $2x + y$  subject to the constraints  $x + y \leq 10$ ,  $y \geq 2$ ,  $x \geq 3$  and  $2x + 3y \leq 24$ .



$$\begin{aligned} A: x=3, y=2 & & 2x+y=8 \\ B: x=3, 2x+3y=24 \Rightarrow 6+3y=24 \Rightarrow y=6 & & 2x+y=12 \\ C: x+y=10, 2x+3y=24 \Rightarrow y=4 \Rightarrow x=6 & & 2x+y=16 \\ D: y=2, x+y=10 \Rightarrow x=8 & & 2x+y=18 \end{aligned}$$

$$x = \underline{8} \quad y = \underline{2}$$

8. a) What is meant by confounding of factors in an experiment?

**Several different variables all changing the outcome so that the influence of one variable cannot be distinguished.**

b) What is meant by a double-blind trial? Explain why this is useful.

**An experiment in which both the patient and the doctor do not know who is receiving the medicine and who is receiving the placebo. This is useful since it eliminates the confounding effect of the attitudes of the patient and doctor to the medicine.**

- c) In setting up a new curriculum, a school district invites families to volunteer. These children are then taught using a new curriculum, while the remaining children are taught using the old curriculum. Their results are compared at the end of two years. Describe any problems which you see with this study and suggest methods to correct them.

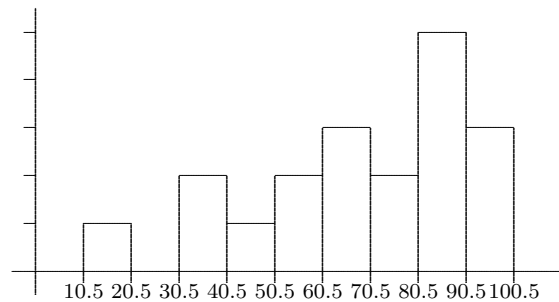
**The main problem is that the children selected for the new curriculum are selected by their parents, and so may not be a representative sample. The children should be divided between the new and old courses randomly.**

9. The scores on a test for a class are:

67, 35, 82, 90, 40, 51, 63, 100, 82, 74, 85, 19, 60, 75, 81, 93, 98, 66, 50.

- a) Choosing suitable ranges, draw a histogram of the data.

Class	Frequency
11-20	1
21-30	0
31-40	2
41-50	1
51-60	2
61-70	3
71-80	2
81-90	5
91-100	3



- b) Describe the features of the data that you observe.

**Not symmetric, skewed to left with one outlier (19)**

- c) Do you think the mean or the median would be larger? Why?

**Median. Since it is skewed to left the mean will be lower.**

- d) Calculate the mean and median for the data and verify your answer to c).

$$\text{Mean} = \underline{69} \quad \text{Median} = \underline{74}$$

10. Five students from a class of 80 are to be picked at random to conduct a survey. Give a good method of selecting these students given the following table of random digits. 36948 00783 93515 10801 00626 54898 85558 97708

**Assign 2-digit numbers from 01 to 80 to the students.**

**Pick students with numbers 36, 80, 07, 51, 08.**

11. a) A combination lock consists of four digits. If all combinations are equally likely, what is the probability that a lock has a combination with all its digits distinct?

$$\#S = 10 \times 10 \times 10 \times 10 = 10000. \quad \#E = 10 \times 9 \times 8 \times 7 = 5040.$$

$$\text{Probability } 5040/10000 = 0.504$$

- b) An urn contains 3 red balls and 2 green balls. Two balls are removed. What is the probability that both are red?

$$\#S = 5 \times 4 = 20. \quad \#E = 3 \times 2 = 6. \quad \text{Probability } 6/20 = 0.3$$

12. In a factory, the mean and standard deviation weights of the contents of a box of cornflakes are 504g and 20g. The boxes are labelled 500g. An inspection of 100 boxes of corn flakes is done. If the average weight of the inspected boxes is less than 500g, the company is fined \$1000.

- a) What can you say about the distribution of the average weight? **Normal**  
 b) What is the mean value of the average weight? **504g**  
 c) What is the standard deviation of the average weight?  $\frac{20}{\sqrt{100}} = \frac{20}{10} = 2g$   
 b) What is the probability that the company  
**Fined if less than 500g which is 2 standard deviations less than mean.**  
**Probability 2.5%**

13. What is the correct check digit for the ISBN number 0-6502-9101-?. [ISBN numbers use weights 10, 9, 8, 7, 6, 5, 4, 3, 2, 1 respectively for the digits and modulus 11.]

$$10(0) + 9(6) + 8(5) + 7(0) + 6(2) + 5(9) + 4(1) + 3(0) + 2(1) + 1(x) = 157 + x. \text{ Must be divisible by 11, so } x = 8.$$

14. Consider the binary linear code shown here with 4 digits + 3 check digits. Each row gives weights, and each weighted sum should be even. For example, the message 0110 is translated as 0110011.

$a_1$	$a_2$	$a_3$	$a_4$	$c_1$	$c_2$	$c_3$
1	1	1	0	1	0	0
1	0	1	1	0	1	0
1	1	0	1	0	0	1

- a) Find the code for 1010. **1010001**  
 b) A code is given as 1111010. Assuming at most one error occurred, what was the original 4-digit number? **1011**  
 c) The weight of this code is 3. How many errors can this code  
 Detect? 2 Correct? 1

15. Gravity accelerates a falling object so that after 1 second it is falling at 32 feet per second. How much is this in miles per hour? [1 mile = 5280 ft].

$$32 \frac{1 \text{ ft}}{1 \text{ s}} = 32 \frac{(1/5280) \text{ mile}}{(1/3600) \text{ hr}} = 21.8 \text{ mph}$$

16. To make a cardboard box holding 25 cubic feet requires 50 square feet of cardboard. How much cardboard would you need for a box holding 200 cubic feet?

**If linear scaling factor is L then  $200 = 25L^3$ ,  $L^3 = 8$ ,  $L = 8^{1/3} = 2$ . Area of cardboard is  $50L^2 = 50(2)^2 = 200$  square feet.**

17. If a quantity increases by 20% and then increases again by 30% again, by how much has it increased in total?

**If quantity is Q we get  $(1 + \frac{30}{100})(1 + \frac{20}{100})Q = (1.30)(1.20)Q = 1.56Q = (1 + \frac{56}{100})Q$ . Thus quantity increases 56%.**

18. Give the symmetry type (e.g., d4, c5) for each of the following figures and draw on each diagram all lines of reflectional symmetry (if there are any).

